E 2208

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B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, MAY 2018

Second Semester

Complementary Course—Mathematics

INTEGRAL CALCULUS AND MATRICES

(Common for B.Sc. Physics, Chemistry, Petrochemicals, Geology, Food Science and Quality Control and Computer Maintenance and Electronics)

[2013—2016 Admissions]

Time : Three Hours

Maximum Marks : 80

Part A

*Answer **all** questions.*
Each question carries 1 mark.

1. State fundamental theorem of calculus.

2. Find $\int_{-2}^{-1} x \, dx$.

3. Write the formula for finding the length of a plane curve $y = f(x)$ on the interval $[a, b]$.

4. Write the formula for finding the surface area of the surface of revolution that is generated by revolving the portion of the curve $y = f(x)$, between $x = a$, $x = b$ about x-axis.

5. Write the formula for the volume of a solid of revolution about y-axis.

6. Find $\int_0^3 \int_1^2 (1 + 8xy) \, dy \, dx$.

7. Write the formula for finding the area of a region in xy plane in terms of double integrals.

8. State Cayley-Hamilton theorem.

9. Define a non-singular matrix.

10. Define rank of a matrix.

$(10 \times 1 = 10)$

Turn over

Part B

*Answer any **eight** questions.*
Each question carries 2 marks.

11. Evaluate $\int_0^{x^2} \cos \sqrt{t} \, dt.$

12. Evaluate $\int \frac{\log x}{x} \, dx.$

13. Find the area bounded by $xy = c^2$, the x-axis and $x = 2, x = 3$.

14. Find the length of the arc of the curve $y = \log \sec x$ from $x = 0$ to $x = \pi/3$.

15. Determine the volume of the solid of revolution when arc of the curve $y = xe^x$ between $x = 0$ and $x = 1$ is revolved about the line $y = 0$.

16. Evaluate $\int_0^4 \int_0^{x^2} e^{y/x} \, dy \, dx.$

17. Evaluate $\iint r \, dr \, d\theta$ over the area of the cardiode $r = a (1 - \cos \theta)$ above the initial line.

18. Evaluate $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y} \, dy \, dx$ by changing the order of integration.

19. Evaluate $\int_0^3 \int_0^2 \int_0^1 (x + y + z) \, dz \, dx \, dy.$

20. What are the elementary transformations of a matrix.

21. Find the eigen values of the matrix $\begin{bmatrix} 3 & 4 \\ 5 & 2 \end{bmatrix}.$

22. Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \end{bmatrix}.$

$(8 \times 2 = 16)$

Part C

*Answer any **six** questions.*
Each question carries 4 marks.

23. Evaluate $\int\limits_{0}^{1} x \log(1+x)\, dx$.

24. Evaluate $\int\limits_{0}^{\pi/2} \dfrac{\sin x}{\sin x + \cos x}\, dx$.

25. Find the area enclosed by the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$.

26. Find the area enclosed by the parabolas $y^2 = x$ and $x^2 = y$.

27. Find the entire length of the astroid $x^{2/3} + y^{2/3} = a^{2/3}$.

28. Find the volume of the solid that results when the region enclosed by $y = x^2$, $x = 0$, $x = 2$, $y = 0$ is revolved about the $x =$ axis.

29. Evaluate $\iint xy\, dx\, dy$ over the positive quadrant $x^2 + y^2 = a^2$.

30. Obtain the row equivalent canonical matrix of $\begin{bmatrix} 1 & 1 & 1 & 2 \\ 2 & 1 & -3 & -6 \\ 3 & -3 & 1 & 2 \end{bmatrix}$.

31. Show that if λ is a characteristic root of a non-singular matrix A, then λ^{-1} is a characteristic root of A^{-1}.

$(6 \times 4 = 24)$

Part D

*Answer any **two** questions.*
Each question carries 15 marks.

32. (a) Find the area of the segment cut-off from $y^2 = 4x$ by the line $y = x$.

    (b) Find the length of the curve $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$.

Turn over

33. (a) Derive the formula for the volume of a sphere of radius r.

    (b) Find the area of the surface that is generated by revolving the portion of the curve $y = x^2$ between $x = 1$ and $x = 2$ about the y-axis.

34. (a) Find the volume bounded by the xy plane, the cylinder $x^2 + y^2 = 1$ and the plane $x + y + z = 3$.

    (b) Evaluate $\iiint \left(x^2 + y^2 + z^2 \right) dx\, dy\, dz$ bounded by the co-ordinate planes $x = a,\ y = a,\ z = a$.

35. (a) Solve the system of equations :

    $$5x + 3y + 3z = 48$$

    $$2x + 6y - 3z = 18$$

    $$8x - 3y + 2z = 21$$

    (b) Find the eigen values and corresponding eigen vectors of the matrix :

    $$\begin{bmatrix} 5 & 1 & -1 \\ 1 & 3 & -1 \\ -1 & -1 & 3 \end{bmatrix}.$$

    $(2 \times 15 = 30)$