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Reg. No..................................

Name.....................................

M.Sc. DEGREE (C.S.S.) EXAMINATION, MARCH 2015

First Semester

Faculty of Science

Branch I (A)—Mathematics

MT 01 C05—COMPLEX ANALYSIS

(2012 Admissions)

Time : Three Hours

Maximum Weight : 30

Part A (Short Answer Questions)

Answer any **five** *questions.*
Each question has 1 *weight.*

1. Differentiate between bounded and totally bounded sets with examples.
2. Obtain the complex form of Cauchy-Riemann equation.
3. Compute $\int_{|z|=2} \frac{dz}{z^2-1}$ for the positive sense of the circle.
4. Compute $\int_{|z|=2} \frac{dz}{z^2+1}$ by decomposition of the integrand in partial fractions.
5. Differentiate between simply connected and multiply connected regions with examples.
6. Establish the maximum principle.
7. Find the poles are residues of cot z.
8. State and prove Residue Theorem.

$(5 \times 1 = 5)$

Part B (Short Essay Questions)

Answer any **five** *questions.*
Each question has 2 *weight.*

9. Show that every linear transformation which transforms the real axis into itself can be written with red coefficients.
10. Explain the notion of Riemann surface with two examples.

Turn over

11. Obtain representation formula.

12. If $p(z)$ is a polynomial and C is the circle $|z-a|=R$ find the value of $\int_C p(z)\,dz$.

13. Show that any function which is meromorphic in the extended plane is rational.

14. Define with examples : (a) Zero of order h ; (b) Isolated zeros ; (c) Removable singularity.

15. Explain the geometric interpretation of Poisson's formula.

16. How many roots of the equation $z^4+8z^3+3z^2+8z+3=0$ lie in the right half plane ?

(5 × 2 = 10)

Part C (Long Essay Questions)

Answer any **three** *questions.*
Each question has 5 weight.

17. (a) Show that the cross ratio is invariant under linear transformations.

(b) Find necessary and sufficient condition for a cross ratio to be real.

(c) Show that a linear transformation carries circles into circles.

18. (a) Compute $\displaystyle\int_{|z|=p} \frac{|dz|}{|z-a|^2}$ under the condition $|a| \neq p$.

(b) If $f(z)$ is analytic and $|f(z)| \leq M$ for $|z| \leq R$, find an upper bound for $\left|f^{(h)}(z)\right|$ in $|z| \leq P < R$.

19. (a) Obtain Cauchy's integral formula for higher derivatives by method of induction.

(b) Obtain Cauchy's estimate.

(c) State and prove Liouville's theorem.

20. (a) State and prove Taylor's theorem.

(b) Obtain a formula for number of zeros.

(c) Give *two* examples of functions having essential singularity at ∞.

21. Evaluate $\int_0^{\pi} \log \sin \theta \, d_{\theta}$.

22. Evaluate $\displaystyle\int_0^{\infty} \frac{x^2}{\left(x^2+a^2\right)^2}\,dx$, a real.

(3 × 5 = 15)