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Reg. No.....

Name.....

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, MARCH 2023

First Semester

Complementary Course—DIFFERENTIAL CALCULUS AND TRIGONOMETRY

(Complementary Course for Physics/Chemistry/Petrochemicals/Geology/Food Science and Quality Control/Computer Maintenance and Electronics)

[2013—2016 Admissions]

Time : Three Hours Maximum Marks : 80

Part A

Answer **all** *questions.*
Each question carries 1 *mark.*

1. Find $\lim_{x \to -3} \frac{x+3}{x^2+4x+3}$.
2. State quotient rule of differentiation.
3. Find $\lim_{y \to 0} \frac{\sin 3y}{4y}$.
4. State first derivative theorem for local extreme values.
5. State Rolle's theorem.
6. Find the domain where the function $f(x) = x^2$ increases.
7. If $f(x, y) = x^2 - xy + y^2$, find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$.
8. Write Euler's theorem for partial derivatives.
9. Write sin (ix) and cos (ix) in terms of hyperbolic functions.
10. Write tanh x in terms of e^x.

(10 × 1 = 10)

Part B

Answer any **eight** *questions.*
Each question carries 2 *marks.*

11. Discuss the behaviour of $g(x) = \begin{cases} \frac{1}{x}, & x \neq 0 \text{ as } x \to 0. \\ 0, & x = 0 \end{cases}$
12. Find the tangent line to the curve $f(x) = \frac{8}{\sqrt{x-2}}$ at (6, 4).

Turn over

13. Find $\frac{dr}{d\theta}$ when $r = 4 - \theta^2 \sin\theta$.

14. Find the absolute maximum and minimum values of $f(x) = x^2 - 1, -1 \le x \le 2$.

15. Verify mean value theorem for $f(x) = x^2 + 2x - 1$ on [0, 1].

16. Find the values of any local maxima and minima for $f(x) = x^2 - 4$ on $-2 \le x \le 2$.

17. Find $\frac{\partial f}{\partial y}$ if $f(x, y) = y\sin(xy)$.

18. Find $\frac{dy}{dx}$ at (1, 1), given $x^2 - 2y^2 + xy = 0$.

19. Verify that $wxy = wyx$ for $w = \log(2x + 3y)$.

20. Separate $\cos(x - iy)$ into real and imaginary parts.

21. If $\sin(A + iB) = x + iy$ show that $\frac{x^2}{\sin^2 A} - \frac{y^2}{\cos^2 A} = 1$.

22. Prove that $\cos 4\theta = \cos^4\theta - 6\sin^2\theta\cos^2\theta + \sin^4\theta$.

$(8 \times 2 = 16)$

Part C

Answer any **six** *questions.*
Each question carries 4 *marks.*

23. At time to the position of a body moving along the s-axis is $s = t^3 - 6t^2 + 9t$ *m.* (a) Find the body is acceleration each time the velocity is zero ; (b) Find the body's speed each time the acceleration is zero.

24. (a) Evaluate $\lim_{t\to 0} \frac{\sin(1-\cos t)}{1-\cos t}$.

    (b) Find $\frac{dp}{dq}$ if $p = \frac{\tan q}{1 + \tan q}$.

25. Find the derivative of $g(t) = \tan(s - \sin 2t)$.

26. Find the absolute extreme values of $f(x) = \sqrt{x} + \cos x$ on $[0, 2\pi]$.

27. For what values of a, m and b does the function :

$$f(x)=\begin{cases}3, & x=0\\ -x^2+3x+a, & 0<x<1\\ mx+b, & 1\le x\le 2\end{cases}$$

Satisfy the hypothesis of mean value theorem on [0, 2].

28. Find all second order partial derivative of $h(x,y)=xe^y+y+1.$.

29. Find $\frac{dw}{dt}$ if $w=xy+z, x=\cos t,\ y=\sin t, z=t$. What is the vlaue of derivative at $t=0$?

30. Prove that $\sin^5\theta=\frac{1}{16}[\sin 5\theta-5\sin 3\theta+10\sin\theta]$.

31. If $\sin(\theta+i\phi)=\tan(x+iy)$, show that $\frac{\tan\theta}{\tanh\theta}=\frac{\sin 2x}{\sinh 2y}$.

(6 × 4 = 24)

Part D

Answer any **two** *questions.*
Each question carries 15 *marks.*

32. (a) If $\sqrt{5-2x^2}\le f(x)\le\sqrt{5-x^2}$ for $-1\le x\le 1$, find $\lim_{x\to 0} f(x)$.

(b) Discuss the behaviour of $g(x)=\frac{1}{(x+3)^2}$ near $x=-3$.

(c) Find $\frac{d^2y}{dx^2}$ if $2y=x^2+\sin y$.

33. (a) Given that $f'(x)=x(x-1)$.

(i) What are the critical points of f.

(ii) On what interval is f increasing or decreasing.

(iii) At what points, if any, does f assumes local maximum and minimum values.

(b) Find the velocity and displacement functions of a body falling freely from rest with acceleration 9.8 m./sec.^{2}

Turn over

34. (a) Find f_x, f_y and f_z given $f(x, y, z) = \tanh(x + 2y + 3z)$.

(b) Using chain rule find $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}$ and $\frac{\partial u}{\partial z}$ at (x, y, z) given

$$u = \frac{p - q}{q - r},\ p = x + y + z,\ q = x - y - z,$$

$$r = x + y - z \text{ at } (x, y, z) = \left(\sqrt{3}, 2, 1\right).$$

35. (a) Sum to infinity the series $c \sin\alpha + \frac{1}{2}c^2 \sin 2\alpha + \frac{1}{3}c^3 \sin 3\alpha + \ldots, 1 < 1 < 1.$

(b) Express $\cos^5\theta \sin^4\theta$ in terms of cosines of multiples of θ.

$(2 \times 15 = 30)$