F 5535 (Pages : 2) Reg. No...............................

Name...................................

M.Sc. DEGREE (C.S.S.) EXAMINATION, FEBRUARY 2016

First Semester

Faculty of Science

Branch : I (A) Mathematics

MT 01 C03—MEASURE THEORY AND INTEGRATION

(2012 Admission onwards)

Time : Three Hours Maximum Weight : 30

Section A

Answer any **five** *questions.*
Each question has 1 weight.

1. Define counting set function and show that it is countably additive and translation invariant.
2. Show that linear combination of measurable functions is also measurable.
3. Prove that Dirichlet's function is not Riemann integrable.
4. If f is integrable show that $|f|$ is also integrable. Is the converse true ? Explain.
5. Differentiate between measurable space and measure space with example.
6. If f is a non-negative measurable function on a measure space show that $\int f\,d\mu = 0$ iff $f = 0$ *a.e.*
7. Show that if $f_n \to f$ in measure than $|f_n| \to |f|$ in measure.
8. Show that the product of complete measures need not be a complete measure.

(5 × 1 = 5)

Section B (Short Essay Type Questions)

Answer any **five** *questions.*
Each question has 2 weight.

9. Prove that every interval is a borel set and the translate of a measurable set is measurable.
10. Define measurable set with a non-trivial example. Prove your assertion.
11. Give an example to show that point wise convergence alone is not sufficient to justify passage of the limit under integral sign.
12. Compute the upper and lower derivatives of the characteristic function of the rationals.

Turn over

13. Differentiate between counting measure and Dirac measure. Also differentiate between measure and signed measure.

14. Obtain necessary and sufficient condition for the extended real valued function to be measurable.

15. If $f_n \to f$ *a.u.* prove

    (a) $f_n \to f$ in measure

    (b) $f_n \to f$ *a.e.*

16. By integrating $e^{-y} \sin 2xy$ with respect to x and y show that :

$$\int_0^\infty e^{-y} (\sin^2 y)/y \, dy = \frac{1}{4} \log 5.$$

(5 × 2 = 10)

Section C (Long Essay Type)

Answer any **three** *questions.*
Each question has 5 *weight.*

17. (a) Establish Vitali's theorem on non-measurable set.

    (b) Let f be an extended real-valued function on E. If f is measurable on E and $f = g$ a.e. on E. Show that g is measurable on E.

18. (a) State and prove bounded convergence theorem.

    (b) State and prove the monotone convergence theorem and prove that it may not hold for decreasing sequences of functions.

19. (a) State and prove Lebesgue convergence theorem for measurable functions.

    (b) Let f be a bounded function defined on a measurable set E with $mE < \infty$. Obtain necessary and sufficient condition for f be measurable.

20. Let (X, M) be a measurable space and f and g measurable real-valued function on X. Prove (i) linearity ; (ii) products (iii) Maximum and minimum properties. Also prove that measurability of functions is preserved under the formation of pointwise limits.

21. Establish the Radon-Nikodyn theorem.

22. (a) Establish Fubini's theorem.

    (b) Give example to show that sigma-finiteness is essential in Fubini's theorem.

(3 × 5 = 15)