F 7891 (Pages : 2) Reg. No..............................

Name..................................

M.Sc. DEGREE EXAMINATION, FEBRUARY 2021

Fourth Semester

Faculty of Science

Branch I—(a)—Mathematics

Elective Paper—PROBABILITY THEORY

(For Private Registration Candidates)

[Regular/Supplementary/Mercy Chance and Non-CSS College Going (2004–2011) Admissions Special Mercy Chance Examination]

Time : Three Hours Maximum : 75 Marks

Answer **either** *Part (A)* **or** *Part (B) only of each question.*
Each question carries 15 *marks.*

I. A (a) Define (i) Minimal sigma field ; (ii) Borel sigma field ; (iii) Limit inferior ; (iv) Probability space.

(4 marks)

(b) Show that the intersection of an arbitrary number of sigma field is a sigma field.

(5 marks)

(c) Explain axiomatic definition of probability and state its properties. (6 marks)

B (a) Prove that a sigma field is a monotone field and conversely. (5 marks)

(b) Prove that any Borel function of a vector random variable (X, Y) is a random variable.

(6 marks)

(c) Define (i) Lebesgue Measure ; (ii) Signed. (4 marks)

II. A (a) Let X and Y be two random variables with joint bivariate normal distribution with parameters $\mu_1, \mu_2, \sigma_1, \sigma_2$ and p then find the marginal pdf of X.

(6 marks)

(b) Let a random variable X have the distribution

$P(X = 0) = P(X = 2) = p$. P(X = 1) = 1 – 2*p* for $0 \leq p \leq \frac{1}{2}$. For what value of *p* the variance is maximum.

(4 marks)

(c) Define moment generating function. Find mgf of Gamma distribution with parameter λ and p.

(5 marks)

Turn over

B (a) State and prove Jordan Decomposition Theorem. (5 marks)

(b) State and establish the three properties of expectation of simple random variables. (6 marks)

(c) Let X and Y be two random variables having finite means. Prove that $E[\text{Min}(x, y)] \le \text{Min}[E(x) \cdot E(y)]$. (4 marks)

III. A (a) If $X_n \xrightarrow{P} X$ and $Y_n \xrightarrow{P} Y$, then prove that $aX_n \xrightarrow{P} aX$, where a is a real number. (5 marks)

(b) Define characteristic function. Find the characteristic function of the exponential distribution with pdf $f(x) = e^{-x}, \quad x \ge 0$
$= 0 \quad$ elsewhere
and hence obtain the first moment. (10 marks)

B (a) State and prove Fatou's theorem. (10 marks)

(b) For any characteristic function ϕ. Prove that $\text{Re}[1-\phi(u)] \ge \frac{1}{4}\text{Re}[1-\phi(24)]$ where $\text{Re}(\phi)$ is the real part of ϕ. (5 marks)

IV. A (a) State and prove Second Limit Theorem. (9 marks)

(b) Explain mutual independence of random variables. Prove by an example that total independence need not imply mutual independence. (6 marks)

B (a) Explain weak and complete convergence of distribution function. (6 marks)

(b) Defne independence of classes. Prove that subclass of independent classes are independent. (9 marks)

V. A (a) State and prove Kolmogorov inequality. (8 marks)

(b) State Lindberg-Feller form of CLT. One thousand rounds are fired from a gun at a target. The probability of a hit on each round is 0.7. Use CLT to determine the probability that the number of hit will be between 680 and 720. (7 marks)

B (a) State and prove Kronecker's Lemma. (7 marks)

(b) Let $\{X_K\}$ be mutually independent and identically distributed random variables with mean μ and finite variance σ^2. If $S_n = X_1 + X_2 + \ldots + X_n$, prove that weak law of large numbers does not hold for $\{S_n\}$. (8 marks)

[5 × 15 = 75 marks]