E 4217 (Pages : 4) Reg. No.....

Name.....

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, NOVEMBER 2016

First Semester

Complementary Course—DIFFERENTIAL CALCULUS AND TRIGONOMETRY

(Complementary Course for Physics/Chemistry/Petrochemicals/Geology/Food Science and Quality Control/Computer Maintenance and Electronics)

[2013 Admission onwards]

Time : Three Hours Maximum Marks : 80

Part A (Short Answer Questions)

Answer **all** *questions.*
Each question carries 1 *mark.*

1. State the Sandwich theorem.
2. Define the derivative of a function f at $x = x_0$.
3. State the derivative quotient rule.
4. State the first derivative theorem for local extreme values.
5. Give a physical interpretation of mean value theorem.
6. Define a decreasing function.
7. Define the partial derivative of $f(x, y)$ with respect to x at (x_0, y_0).
8. State chain rule for functions of three independent variables.
9. Show that $\cos(y\,i) = \cosh y$.
10. What is the period of $\sinh(x + yi)$?

$(10 \times 1 = 10)$

Part B (Brief Answer Questions)

Answer any **eight** *questions.*
Each question carries 2 *marks.*

11. Prove that $\lim_{x \to x_0} k = k$ (k constant).
12. Find the slope of the tangent to the curve $y = x + \frac{2}{x}$ at the point (1, 3).

Turn over

13. Find $\frac{dy}{dx}$ if $x = \tan y$.

14. Find the absolute extreme of $f(x) = -x - 4$ in $-4 \le x \le 1$.

15. Show that the equation $x^3 + 3x + 1 = 0$ has a real solution.

16. If $f'(x) = x(x-1)$, find the intervals on which f is increasing and decreasing.

17. Find the partial derivatives f_x, f_y and f_z if $f(x, y, z) = x - \sqrt{y^2 + z^2}$.

18. Find $\frac{dw}{dt}$ at $t = 0$ if $w = x^2 + y^2$, $x = \cos t$, $y = \sin t$.

19. Find $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$ if $f(x, y) = e^x \sin y$.

20. Prove that $\cos 4\theta = \cos^4\theta - 6\cos^2\theta \sin^2\theta + \sin^4\theta$.

21. Prove that $\cosh(\alpha + \beta) = \cosh\alpha \cosh\beta + \sinh\alpha \sinh\beta$.

22. Prove that $\cosh^{-1} x = \log\left(x + \sqrt{x^2 - 1}\right)$.

(8 × 2 = 16)

Part C (Descriptive/Short Essay Type Questions)

Answer any **six** *questions.*
Each question carries 4 *marks.*

23. Find (i) $\lim_{y \to 0} \frac{\sin 3y}{4y}$. (ii) $\lim_{x \to \infty} \frac{x+1}{x^2+3}$.

24. Show that $f(x) = |x|$ is not differentiable at the origin.

25. Find $\frac{d^2y}{dx^2}$ as a function of t if $x = t + \frac{1}{t}$, $y = t - \frac{1}{t}$.

26. State and prove Rolle's theorem.

27. Find the critical points of $f(x) = x^3 - 12x - 5$ and identify the intervals on which f is increasing and decreasing.

28. Verify that $w_{xy} = w_{yx}$ if $w = xy^2 + x^2 y^3 + x^3 y^4$.

29. Express $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial s}$ in terms of r and s if $w = x + 2y + z^2$, $x = \frac{r}{s}$, $y = r^2 + \ln s$, $z = 2r$.

30. Expand $\sin^6\theta$ in a series of cosines of multiples of θ.

31. If $\sin(A + iB) = x + iy$, prove that $\frac{x^2}{\cosh^2 B} + \frac{y^2}{\sinh^2 B} = 1$ and $\frac{x^2}{\sin^2 A} - \frac{y^2}{\cos^2 A} = 1$.

(6 × 4 = 24)

Part D (Long Essay Questions)

Answer any **two** *questions.*
Each question carries 15 *marks.*

32. (a) Prove that limit statement $\lim_{x \to 4} (9 - x) = 5$.

(b) Does the curve $y = x^4 - 2x^2 + 2$ have any horizontal tangents ? If so, where ?

(c) If $x^3 + y^3 = 16$, find the value of $\frac{d^2y}{dx^2}$ at the point (2, 2).

33. (a) Find the point c of the mean value theorem for the function $f(x) = x^3 - x$ in $0 \le x \le 2$.

(b) If $f'(x) = 0$ for all x in an open interval (a, b), prove that f is constant on (a, b).

(c) Find the critical points of $f(x) = x^{1/3}(x - 4)$ and identify the intervals on which f is increasing and decreasing.

34. (a) If resistors R_1, R_2 and R_3 ohms are connected in parallel to make an R-ohm resistor, the value of R can be found from the equation $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$.

Find the value of $\frac{\partial R}{\partial R_2}$ when $R_1 = 30$, $R_2 = 45$ and $R_3 = 90$ ohms.

(b) Find the value of $\frac{\partial z}{\partial x}$ at the point (1, 1, 1) if the equation $xy + z^3x - 2yz = 0$ defines z as a function of two independent variables x and y and the partial derivative exists.

(c) State a formula for implicit differentiation. Use it to find $\frac{dy}{dx}$ at (1, 2) if $x^2 + xy + y^2 = 7$.

35. (a) Separate into real and imaginary parts of $\tan^{-1}(\alpha + \beta i)$.

(b) Find the sum to infinity of the series $\sin\alpha + \frac{1}{2}\sin 2\alpha + \frac{1}{2^2}\sin 3\alpha + \ldots$

(c) Sum to infinity the series $c\cos\alpha + \frac{c^2}{2}\cos 2\alpha + \frac{c^3}{3}\cos 3\alpha + \ldots$, where $|c| < 1$.

(2 × 15 = 30)