G 3503 (Pages : 2) Reg. No..............................

Name..................................

M.Sc. DEGREE (C.S.S.) EXAMINATION, AUGUST 2015

Second Semester

Faculty of Science

Branch I (a) : Mathematics

MT 02 108—ADVANCED COMPLEX ANALYSIS

(2012 Admission onwards)

Time : Three Hours Maximum Weight : 30

Part A

Answer any **five** *questions.*
Each question has weight 1.

1. Define meromorphic function with an example. Explain.
2. If the radius of convergence of $\sum a_n z^n$ is R. Find the radii of convergence of $\sum a_n^2 z^n$ and $\sum a_n z^{2n}$.
3. Define : (i) entire function ; and (ii) genus and order.
4. Extend the Riemann Zeta function to a meromorphic function in the whole plane.
5. Explain : (i) free boundary arc ; and (ii) topological mapping.
6. Define : subharmonic function with example. Explain.
7. Define elliptic function. Prove that the sum of the residues of an elliptic function is zero.
8. Explain functions of finite order with illustrations.

$(5 \times 1 = 5)$

Part B

Answer any **five** *questions.*
Each question has weight 2.

9. State and prove Abel's limit theorem.
10. Prove that the Laurent development is unique.
11. Characterise a normal family using compactness.
12. Reproduce the proof of functional equation.
13. Characterise subharmonic functions using an inequality which generalizes the mean value property of harmonic functions.
14. Define functions with the mean value property. Show that a continuous function with mean value property is necessarily harmonic.

Turn over

15. Define module, period module and discrete module. Explain how to determine all discrete modules.

16. Derive Legendre's relation.

(5 × 2 = 10)

Part C

Answer any **three** *questions.*
Each question has weight 5.

17. (a) Derive a recurrence formula for Gamma functions.

(b) Find the residues of Γz at the poles $z = -\infty$.

(c) Obtain the Legendre's duplication formula for Gamma functions.

18. (a) If $d(a, b)$ is a metric function, show that $\delta(a, b) = \dfrac{d(a, b)}{1 + d(a, b)}$ is also a metric function.

(b) Show that there are infinitely many primes.

(c) Explain the concept of equi-continuity.

19. (a) Obtain Harnack's inequality.

(b) Derive Harnack's principle.

20. State and prove the Riemann mapping theorem.

21. State and prove the theorem on the existence of Canonical basis.

22. With usual notation prove $\begin{vmatrix} \wp(z) & \wp'(z) & 1 \\ \wp(u) & \wp'(u) & 1 \\ \wp(u+z) & -\wp'(u+z) & 1 \end{vmatrix} = 0.$

(3 × 5 = 15)