QP CODE: 21000381

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Reg No : ....

Name : ....

M Sc DEGREE (CSS) EXAMINATION, MARCH 2021

Third Semester

Faculty of Science

CORE - ME010301 - ADVANCED COMPLEX ANALYSIS

M Sc MATHEMATICS,M Sc MATHEMATICS (SF)

2019 Admission Onwards

20025F71

Time: 3 Hours | Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. Prove that $log r$ is a harmonic function.
2. Show that Poisson integral is a linear functional.
3. Find the Laurent series expansion of $f(z) = \frac{1}{z(z-1)^2}$ about the point $z = 1$.
4. Find the poles of $\frac{\pi^2}{sin^2 \pi z}$ and the corresponding singular parts.
5. Prove that $\Gamma(z+1) = z\Gamma(z)$.
6. *Prove that the zeta function can be extended to a meromorphic function in the whole plane whose only pole is a simple pole at s=1 with residue 1.*
7. *Prove that* $\xi(s) = \frac{1}{2}s(1-s)\pi^{-(\frac{s}{2})}\Gamma(\frac{s}{2})\zeta(s)$ *is entire.*
8. What do you mean by the trivial zeros of the Riemann zeta function?
9. Define the Riemann mapping.
10. Define a free boundary arc. Give an example.

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. Let Ω be a symmetric region. Prove that $u(z)$ and $u(\bar{z})$ are simultaneously harmonic in Ω. Also prove that $f(z)$ and $f(\bar{z})$are simultaneously analytic in Ω.

12. Prove that *a continuous function* $U(z)$, *which satisfies the mean value property is necessarily harmonic.*

13. Prove that the infinite product $\Pi_1^{\infty}(1 + a_n)$ with $(1 + a_n) \neq 0$ converges simultaneously with the series $\Sigma_1^n log(1 + a_n)$ whose terms represent the values of the principal branch of the logarithm.

14. State and prove Poisson-Jensen's formula.

15. *State and prove a characterisation theorem for a family* $\mathcal{F}$ *of functions to be normal.*

16. *If a family* $\mathcal{F}$ *of continuous functions with values in a metric space S is equicontinuous on every compact subsets of* Ω *and for any* $z \in \Omega, f \in \mathcal{F}$, *the values* $f(z)$ *lie in a compact subset of S, then prove that* $\mathcal{F}$ *is normal in* Ω.

17. Prove that any two bases of the period module are connected by a unimodular transformation.

18. Define the ζ - function. Prove that $\zeta(z) = \frac{1}{z} + \sum_{\omega \neq 0}(\frac{1}{(z-\omega)} + \frac{1}{\omega} + \frac{z}{\omega^2})$

(6×2=12 weightage)

Part C (Essay Type Questions)

*Answer any **two** questions.*

*Weight **5** each.*

19. Derive any four properties of subharmonic functions.

20. (i) Derive the Taylor series for an analytic function $f(z)$ in a region Ω containing the point z_0.
(ii) State and prove Weirstrass's theorem for the convergence of a sequence of analytic functions.
(iii) State and prove Hurwitz theorem.

21. (i) Prove that the Riemann Zeta function is analytic in the half plane $Re\ s > 1$.
(ii) Prove that for $\sigma > 1,\ \zeta(s) = \prod_{p_n, prime} (\frac{1}{1-p_n^s})^{-1}$.
(iii) Prove that for $\sigma > 1,\ \zeta(s) = \frac{1}{\Gamma s}\int_0^{\infty} \frac{x^{s-1}}{e^x - 1} dx$.

22. Prove that $[\wp'(z)]^2 = 4[\wp(z)]^3 - g_2\wp(z) - g_3$, where g_2 and g_3 are constants.

(2×5=10 weightage)