E 7641 (Pages : 3) Reg. No. 1600210490 3

Name. ABIN ROBERT

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, OCTOBER 2017

Third Semester

Complementary Course—Mathematics

VECTOR CALCULUS, DIFFERENTIAL EQUATIONS AND ANALYTIC GEOMETRY

(Common for B.Sc. Physics, Chemistry, Petrochemicals Geology, Computer Maintenance and Electronics and Food Science and Quality Control)

[2013 Admission onwards]

Time : Three Hours Maximum Marks : 80

Part A

Answer **all** *questions.*
Each question carries 1 *mark.*

1. Define continuity of a vector function $r(t)$.
2. Find the length of one turn of the helix $r(t) = \cos t\, i + \sin t\, j + tk$.
3. Define curvature of a curve.
4. Find the gradient field of $f(x, y, z) = xyz$.
5. Define an exact differential form.
6. Define tangential form of Green's theorem.
7. Find an integrating factor of $\left(x^2 - 2x + 2y^2\right)dx + 2xydy = 0$.
8. Write the general form of Clairant's equation.
9. Write the equation of the ellipse where foci : $(0, \pm 3)$, eccentricity : 0.5.
10. Replace the polar equation by equivalent Cartesian equation $r \cos \theta = -4$.

(10 × 1 = 10)

Part B

Answer any **eight** *questions.*
Each question carries 2 *marks.*

11. Show that the curvature of a circle of radius a is $1/a$.
12. Find N for the helix $r(t) = a \cos t\, i + a \sin t\, j + bt\, k, a, b \geq 0, a^2 + b^2 \neq 0$, where N is the principal unit normal.
13. Find the derivative of $f(x, y) = 2xy - 3y^2$ at (5, 5) in the direction of A = $4i + 3j$.

Turn over

14. Evaluate $\int_C (xy+y+z)\,ds$ along the curve $r(t)=2ti+tj+(2-2t)k, 0\le t\le 1$.

15. Find the work done by force $F=\sqrt{z}i-2xj+\sqrt{y}k$ along the straight line path $r(t)=ti+tj+tk, 0\le t\le 1$ from (0, 0, 0) to (1, 1, 1).

16. State Stoke's theorem.

17. Solve $p^3-4xyp+8y^2=0$, where $p=\frac{dy}{dx}$.

18. Solve $(2x+e^y)dx+xe^y dy=0$.

19. Solve $\frac{dy}{dx}+y\tan x=\cos x$.

20. Sketch the parabola $x = 2y^2$.

21. Find the polar equation for the circle $x^2 + (y-3)^2 = 9$.

22. Find the directrix of the parabola $r=\frac{25}{10+10\cos\theta}$.

(8 × 2 = 16)

Part C

Answer any **six** *questions.*
Each question carries 4 marks.

23. Without finding T and N, write the accelaration of the motion :

$$r(t)=(\cos t+t\sin t)i+(\sin t-t\cos t)j,\ t>0$$

in the form $a=a_T T+a_N N$.

24. Find the directions in which $f(x,y)=x^2/2+y^2/2$ increases most rapidly and decreases most rapidly at the point (1, 1).

25. Find the circulation and flux of $F=xi+yj$ around and across the circle $r(t)=\cos t\, i+\sin t j, 0\le t\le 2\pi$.

26. Integrate $g(x, y, z) = y + z$ over the surface of the wedge in the first octant bounded by the co-ordinate planes and the planes $x = 2$ and $y + z = 1$.

27. Verify divergence theorem for the field $F = xi + yj + zk$ over the sphere $x^2 + y^2 + z^2 = a^2$.

28. Solve the equation $y = 2px + y^2 p^3$.

29. Solve $(x^2-3xy+2y^2)dx+x(3x-2y)dy=0$.

30. Derive the standard equation of the ellipse.

31. Sketch the hyperbola $8x^2 - 2y^2 = 16$ include asymptotes and foci.

(6 × 4 = 24)

Part D

Answer any **two** *questions.*
Each question carries 15 *marks.*

32. (a) Find K and T for the helix $r(t) = a\cos t\ i + a\sin t\ j + btk, a, b \geq 0, a^2 + b^2 \neq 0.$

(b) Find the tangent plane and normal line of the surface $f(x, y, z) = x^2 + y^2 + z - 9 = 0$ at the point $p_0(1, 2, 4)$.

33. Use surface integral in Stoke's theorem to calculate the circulation of the field $F = x^2 i + 2xj + z^2 k$ around the ellipse $4x^2 + y^2 = 4$ in the xy plane counter clockwise.

34. Find the net outword flux of the field $F = \dfrac{xi + yj + 2k}{\rho^3}, \rho = \sqrt{x^2 + y^2 + z^2}$ across the boundary of the region D; $0 < a^2 \leq x^2 + y^2 + z^2 \leq b^2$.

35. (a) A wheel of raidus a rolls along a horizontal straight line. Find the parametric equations for the path traced by a point p on the wheel's circumference.

(b) Sketch the conic $r = \dfrac{25}{10 - 5\cos\theta}$.

(2 × 15 = 30)