G 3501 (Pages : 2) Reg. No..............................

Name..............................

M.Sc. DEGREE (C.S.S.) EXAMINATION, AUGUST 2015

Second Semester

Faculty of Science

Branch I (A)—Mathematics

MT 02 C06—ABSTRACT ALGEBRA

(2012 Admission onwards)

Time : Three Hours Maximum Weight : 30

Part A

Answer any **five** *questions.*
Each question carries weight 1.

1. State the fundamental theorem of finitely generated Abelian groups.
2. Define zero of $f(x) \in F[x]$ with an example.
3. Define the two categories of the elements of an extension field with examples.
4. Define finite field with *two* examples.
5. Explain p-group and p-subgroup with examples.
6. Show that every group of prime-power order is solvable.
7. Define splitting field with two examples.
8. Explain : Galois theory gives a beautiful interplay of group and field theory.

$(5 \times 1 = 5)$

Part B

Answer any **five** *questions.*
Each question carries weight 2.

9. Establish the existence of direct product of a finite number of groups.
10. Show-that the multiplicative group of all non-zero elements of a finite field is cyclic.
11. Show that the concept of a field being algebraically closed can be defined in terms of factorization of polynomials over field.
12. Define algebraic extension and finite extension. Show that a finite extension field E of a field F is an algebraic extension of F.
13. Show that there are no simple groups of order $p^r m$ where p is a prime number, r is a positive integer and $m < p$.

Turn over

14. Explain : (i) Burnside's formula ; (ii) Normalizer ; (iii) Sylow p-subgroup ; and (iv) Normal subgroup.

15. Give an example to show that a zero of multiplicity greater than 1 of an irreducible polynomial can occur.

16. Every field of characteristic zero :

    (a) Is perfect—Prove.

    (b) Show that $Q\left(\sqrt[3]{2}\right)$ has only the identity automorphism.

(5 × 2 = 10)

Part C

Answer any **three** *questions.*
Each question carries weight 5.

17. Show that the group $z_m \times z_n$ is cyclic and is :

    (a) Isomorphic to z_{mn} iff gcd of m and n is 1. Extend this to a product of more than 2 factors.

    (b) State and prove division algorithm for F (x).

18. (a) Find all prime numbers p such that $x+2$ is a factor of $x^4+x^3+x^2-x+1$ in $z_p[x]$.

    (b) Characterise a group G to be the interval direct product of subgroups H and K.

19. State and prove the theorem which give us the nature of the field $F(\alpha)$ in case where α is algebraic over F. Give an example illustrating this theorem.

20. (a) Characterise extensions of F of the form $F(\alpha_1; \alpha_2; \ldots\ldots \alpha_n)$ in the case that all the α_i are algebraic over .

    (b) With usual notations prove

    $$[K:F]=[K:E][E:F].$$

21. (a) State and prove Cauchy's theorem on the order a subgroup.

    (b) State the Sylow theorems. Use them to show that no group of order 15 is simple.

22. (a) State and prove the Primitive Element Theorem.

    (b) If E is a finite extension of F prove $\{E:F\}$ divides $[E:F]$.

(3 × 5 = 15)