21000382

QP CODE: 21000382

Reg No : ....

Name : ....

M Sc DEGREE (CSS) EXAMINATION, MARCH 2021

Third Semester

Faculty of Science

CORE - ME010302 - PARTIAL DIFFERENTIAL EQUATIONS

M Sc MATHEMATICS,M Sc MATHEMATICS (SF)

2019 Admission Onwards

82B9F486

Time: 3 Hours   Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. *Verify that the equation* $2y(a-x)\,dx + [z - y^2 + (a-x)^2]\,dy - y\,dz = 0$ *is integrable.*
2. *Form the partial differential equation corresponding to* $x^2 + y^2 + (z-c)^2 = a^2$ *where a and c are arbitrary constants.*
3. Verify that the equation $z = \sqrt{(2x+a)} + \sqrt{2y+b}$ is a complete integral of the partial differential equation $z = \frac{1}{p} + \frac{1}{q}$.
4. Show that the equations $f(x,y,z,p,q) = 0,\ g(x,y,z,p,q) = 0$ are compatible if $\frac{\partial(f,g)}{\partial(x,p)} + \frac{\partial(f,g)}{\partial(y,q)} = 0.$
5. Find a complete integral of the equation $pq = 1.$
6. Prove $F(D,D')e^{ax+by} = F(a,b)e^{ax+by}$.
7. Find the particular integral of $[D^2 - D'^2]z = x - y.$
8. Write the condition for the second order PDE $u_{yy} - yu_{xx} + x^3u = 0$ to be hyperbolic.
9. Prove that $rcos\theta$ satisfy the Laplace's equation , when r, θ, ϕ are spherical polar coordinates
10. Prove that the function $\phi = sinxcoshy + 2cosxsinhy + x^2 - y^2 + 4xy$ is Harmonic.

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. *Find the integral curves of* $\frac{dx}{y(x+y)+az} = \frac{dy}{x(x+y)-az} = \frac{dz}{z(x+y)}$

12. *Find the orthogonal trajectories on the cone* $x^2+y^2=z^2\tan^2\alpha$ *of its intersections with the family of planes parallel to* $z=0$

13. Find the general solution of the linear partial differential equation $x(x^2+3y^2)p - y(3x^2+y^2)q = 2z(y^2-x^2)$.

14. Find the complete integral of the equation $2(z+xp+yq)=yp^2$.

15. Verify that the PDE $z_{xx} - \frac{1}{x}z_x = 4x^2 z_{yy}$ is satisfied by $z=f(x^2-y)+g(x^2+y)$.

16. Solve $\frac{\partial^4 z}{\partial x^4} + \frac{\partial^4 z}{\partial y^4} = 2\frac{\partial^4 z}{\partial x^2 \partial y^2}$.

17. Describe the method of seperation of variables for solving a second order linear partial differential equations.

18. Show that the right circular cones $x^2+y^2=cz^2$ forms a set of equipotential surfaces and show that the corresponding potential function is of the form $A\ log(tan(\theta/2))+B$ where A&B are constants and θ is the usual polar angle.

(6×2=12 weightage)

Part C (Essay Type Questions)

*Answer any **two** questions.*

*Weight **5** each.*

19. a) Prove that if X is a vector such that $X \cdot curlX = 0$ and μ is an arbitrary function of $x, y, z,$ then $(\mu X)\cdot curl(\mu X)=0$.
b) Prove that a necessary and sufficient condition that the Pfaffian differential equation $X \cdot r = 0$ *should be integrable is that* $X \cdot curlX = 0$.

20. Find the general equation of the surfaces orthogonal to the family given by $x(x^2+y^2+z^2)=c_1 y^2$ showing that one such orthogonal set consists of the the family of spheres given by $x^2+y^2+z^2=c_2 z$. If a family exists, orthogonal to both the above equations, show that it must satisfy $2x(x^2-z^2)dx + y(3x^2+y^2-z^2)dy + 2z(2x^2+y^2)dz = 0$.

21. By Jacobi's method, solve $z^2+zu_z-u_x^2-u_y^2=0$.

22. Solve the wave equation $r=t$ by Monge's method.

(2×5=10 weightage)