B.Sc. (PETROCHEMICALS) DEGREE EXAMINATION, FEBRUARY 2017

Second Semester

Mathematics—Paper II

ANALYTICAL GEOMETRY, TRIGONOMETRY AND LINEAR ALGEBRA

(Prior to 2009 Admissions—Mercy Chance)

Time : Three Hours   Maximum : 65 Marks

1. Find all values of $(i)^{113}$.   (3 marks)

2. If w is a non-real cube root of 1, prove that $\dfrac{1}{1+2w} + \dfrac{1}{2+w} - \dfrac{1}{1+w} = 0$.

                   (4 marks)

3. Express $\cos 5\theta$ in terms of $\cos A$.   (5 marks)

4. Expand $\cos^7\theta$ in a series of cosines of multiples of θ.

                   (5 marks)

5. If $\tan \dfrac{x}{2} = \tan h \dfrac{x}{2}$. Show that $\cos x. \cosh x = 1$.

                   (4 marks)

6. Sum the series $\dfrac{\sin\theta}{1!} + \dfrac{\sin 2\theta}{2!} + \dfrac{\sin 3\theta}{3!} + \ldots\ldots \infty$.

                   (4 marks)

7. Sum the series $\cos x - \dfrac{1}{2}\cos 2x + \dfrac{1}{3}\cos 3x + \ldots\ldots + \infty$.

                   (4 marks)

8. Find the condition that $lx + my + n = 0$ to be a tangent to the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$.

                   (5 marks)

9. Find the equation of the pair of tangents from (x_1, y_1) to the parabola $y^2 = 4ax$.

                   (4 marks)

10. Show that the locus of the mid-points of normal chords of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is

$$\left(\frac{x^2}{a^2} - \frac{y^2}{b^2}\right)^2 \left(\frac{a^6}{x^2} - \frac{b^6}{y^2}\right) = (a^2 + b^2)^2.$$

(4 marks)

11. Find the locus of the poles of tangents of the parabola $y^2 = 4ax$ with respect to the circle $x^2 + y^2 = 2ax$.

(4 marks)

12. Show that the product of the perpendiculars from any point of a hyperbola to its asymptotes is constant.

(4 marks)

13. A hyperbola has for its asymptotes the straight lines $2x - y - 3 = 0$ and $3x + y - 7 = 0$ and passes through the point (1, 1). Find its equation. Find also the equation of the conjugate hyperbola.

(5 marks)

14. Prove that the orthocentre of any triangle inscribed in a rectangular hyperbola lies on the curve.

(4 marks)

15. Find the pole of the line $lx + my + 4 = 0$ with respect to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

(4 marks)

16. Find the adjoint of the matrix $\begin{bmatrix} 7 & 6 & 2 \\ -1 & 2 & 4 \\ 3 & 3 & 8 \end{bmatrix}$.

(4 marks)

17. Prove that $\begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$ is orthogonal.

(5 marks)

18. Prove that product of two orthogonal matrices is an orthogonal matrix.

(4 marks)

19. If A and B are square matrices of the same order, prove that adj (AB) = Adj B . adj A.

(4 marks)

20. By reducing to the Echelon form find the rank of the matrix $\begin{bmatrix} 1 & -2 & 1 \\ 2 & 1 & 1 \\ 0 & 5 & -1 \end{bmatrix}$.

(5 marks)

21. Solve the system of equations

$$x + y + z = 9$$
$$2x + 5y + 7z = 52$$
$$2x + y - z = 0.$$

(5 marks)

22. Find the eigen values and corresponding eigen vectors of the matrix

$$\begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}.$$

(5 marks)

23. If $A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$, find A^3 using Cayley-Hamilton theorem.

(5 marks)