F 6494 (Pages : 3) Reg. No..............................

Name..............................

M.Sc. DEGREE (C.S.S.) EXAMINATION, JANUARY 2015

Third Semester

Faculty of Science

Branch I–(A)—Mathematics

MT 03 C11—MULTIVARIATE CALCULUS AND INTEGRAL TRANSFORMS

(2012 Admission onwards)

Time : Three Hours Maximum Weight : 30

Part A

Answer any **five** *questions.*
Each question has weight 1.

1. Write the Fourier series generated by $f \in L([o, p])$ with period p. Also write the formulas for the Fourier co-efficients.
2. Write the inversion formula for Fourier transforms.
3. If $f(x) = \| x \|^2$, then what is the directional derivative $f'(c, u)$ of f. at c in the direction of u.
4. Show by an example that a function can have a finite directional derivative $f'(c, u)$ for every u but may fail to be continuous at c.
5. State the inverse function theorem.
6. If $f = u + iv$ is a complex-valued function with a determinant at a point z in C, then show that $J_f(z) = |f'(z)|^2$.
7. Define a k-form in an open set $E \subset R^n$.
8. State Stoke's theorem.

(5 × 1 = 5)

Part B

Answer any **five** *questions.*
Each question has weight 2.

9. State and prove Weierstrass approximation theorem.
10. Show that $B(p, q) = \dfrac{\Gamma p . \Gamma q}{\Gamma(p+q)}$.

Turn over

11. Let $f : \mathrm{R}^2 \to \mathrm{R}^3$ defined by the equation $f(x, y) = (\sin x \cos y, \sin x \sin y, \cos x \cos y)$. Determine the Jacobian matrix $\mathrm{D}f(x, y)$.

12. Prove that if f is differentiable at c, then f is continuous at c.

13. State and prove mean value theorem for differential calculus.

14. Let A be an open subset of R^n and assume that $f : \mathrm{A} \to \mathrm{R}^n$ has continuous partial derivatives $\mathrm{D}_j f_i$ on A. If $\mathrm{J}_f(x) \neq 0$ for all x in A, prove that f is an open mapping.

15. For every $f \in \zeta\left(\mathrm{T}^{\mathrm{K}}\right)$, prove that $\mathrm{L}(f) = \mathrm{L}'(f)$.

16. Suppose $w = \sum_{\mathrm{I}} b_{\mathrm{I}}(x)\, d\, x_{\mathrm{I}}$ is the standard representation of a k-form w in an open set $\mathrm{E} \subset \mathrm{R}^n$.

Prove that if $w = 0$ in E, then $b_{\mathrm{I}}^{(x)} = 0$ for every increasing k-index I and for every $x \in \mathrm{E}$.

(5 × 2 = 10)

Part C

Answer any **three** *questions.*
Each question has weight 5.

17. State and prove the convolution theorem for Fourier transforms.

18. Assume that g is differentiable at a, with total derivative $g'(a)$. Let $b = g(a)$ and assume that f is differentiable at b with total derivative $f'(b)$. Prove that the composition $h = f \circ g$ is differentiable at a and $h'(a) = f'(b) \circ g'(a)$.

19. Let u and v be two real valued functions defined on a subset S of the complex plane. Assume that u and v are differentiable at an interior point c of S and the partial derivatives satisfy the Cauchy-Riemann equations at c. Prove that the function $f = u + iv$ has derivative at c and $f'(c) = \mathrm{D}_1 u(c) + i\,\mathrm{D}_1 v(c)$.

20. Prove that if both partial derivatives $\mathrm{D}_r f$ and $\mathrm{D}_k f$ exists in an n-ball (c, δ) and if both are differential at c, then $\mathrm{D}_{r,k} f(c) = \mathrm{D}_{k,r} f(c)$.

21. Assume that the second order partial derivatives $\mathrm{D}_{i,j} f$ exist in an n-ball B (a) and are continuous at a, where a is a stationary point of f. Let $\mathrm{Q}(t) = \frac{1}{2} f'(a, t) = \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \mathrm{D}_{i,j} f(a)\, t_i\, t_j$.

Prove that :

(a) If $Q(t) > 0$ for all $t \neq 0$, f has a relative minima at a.

(b) If $Q(t) < 0$ for all $t \neq 0$, f has a relative maxima at a.

(c) If $Q(t)$ takes both positive and negative values, the f has a saddle point at a.

22. Prove the following :—

(a) If w and λ are and k-and m-forms respectively of class $\mathscr{C}'$ in E, then

$$d(w \wedge \lambda) = (d\,w) \wedge \lambda + (-1)^k\, w \wedge d\lambda.$$

(b) If w is of class $\mathscr{C}^n$ in E, then $d^2\, w = 0$. Here E is some open set in R^n.

(3 × 5 = 15)