

QP CODE: 19002355



Reg No : .....

Name : .....

**M.Sc. DEGREE (C.S.S ) EXAMINATION, NOVEMBER 2019**

**First Semester**

Faculty of Science

MATHEMATICS

**Core - ME010104 - REAL ANALYSIS**

2019 Admission Onwards

6A0EF4BF

Time: 3 Hours

Maximum Weight :30

**Part A (Short Answer Questions)**

Answer any **eight** questions.

Weight **1** each.

1. Prove that if  $f$  is monotonic on  $[a, b]$  then  $f$  is of bounded variation on  $[a, b]$ .
2. Prove the additive property of arc lengths.
3. Let  $f(x) = \begin{cases} 0 & \text{if } x \text{ is irrational} \\ 1 & \text{if } x \text{ is rational} \end{cases}$ . Is  $f$  Riemann integrable on  $[0, 1]$ ?
4. If  $f \in \mathcal{R}(\alpha)$  and  $g \in \mathcal{R}(\alpha)$  then show that  $fg \in \mathcal{R}(\alpha)$ .
5. Define the unit step function  $I$ . Is it continuous?
6. If  $\{f_n\}$  and  $\{g_n\}$  are sequences of bounded functions which converge uniformly on a set  $E$ , then prove that  $\{f_n g_n\}$  converges uniformly on  $E$ .
7. State Weierstrass uniform convergence test for series of functions.
8. Under what conditions, a sequence  $\{f_n\}$  of continuous functions defined on a compact set  $K$ , is convergent uniformly to a continuous function  $f$ ?
9. Define pointwise boundedness and uniform boundedness of a sequence of functions.
10. For  $n = 0, 1, 2, \dots$ , and  $x$  real, prove that  $|\sin nx| \leq n|\sin x|$

(8×1=8 weightage)

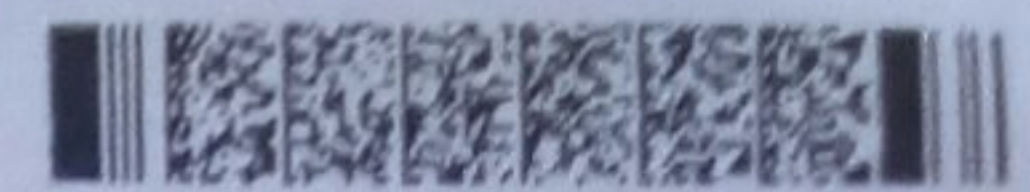
**Part B (Short Essay/Problems)**

Answer any **six** questions.

Weight **2** each.

11. Let  $f$  be defined on  $[a, b]$ . Then show that  $f$  is of bounded variation on  $[a, b]$  if, and only if,  $f$  can be





expressed as the difference of two strictly increasing functions.

12. Prove that a vector valued function  $\mathbf{f}$  is rectifiable if and only if each of its components is of bounded variation.
13. If  $P^*$  is a refinement of  $P$  then prove that  $U(P^*, f, \alpha) \leq U(P, f, \alpha)$ .
14. If  $f$  is monotonic on  $[a, b]$  and if  $\alpha$  is continuous on  $[a, b]$  then prove that  $f \in \mathcal{R}(\alpha)$ .
15. Prove that  $\sum_{n=0}^{\infty} f_n(x)$  is a convergent series having a discontinuous sum, where 
$$f_n(x) = \frac{x^2}{(1+x^2)^n} \quad (x \in \mathbb{R}, n = 0, 1, 2, \dots).$$
16. Let  $\alpha$  be monotonically increasing on  $[a, b]$ . Suppose  $f_n \in \mathcal{R}(\alpha)$  on  $[a, b]$ , for  $n = 1, 2, 3, \dots$  and suppose  $f_n \rightarrow f$  uniformly on  $[a, b]$ . Then prove that  $f \in \mathcal{R}(\alpha)$  on  $[a, b]$ .
17. Let  $K$  be a compact metric space and let  $S$  be a subset of  $\mathcal{C}(K)$ . Prove that  $S$  is compact if and only if  $S$  is uniformly closed, pointwise bounded and equicontinuous.
18. If  $\sum a_n, \sum b_n, \sum c_n$ , converge to  $A, B, C$ , and if  $c_n = a_0 b_n + \dots + a_n b_0$ , prove that  $C = AB$ .  
(6×2=12 weightage)

### Part C (Essay Type Questions)

Answer any **two** questions.

Weight 5 each.

19. (i) State and prove additive property of total variation.  
(ii) Let  $f$  be of bounded variation on  $[a, b]$ . Let  $V$  be defined on  $[a, b]$  as  $V(x) = V_f(a, x)$ , if  $a < x \leq b$  and  $V(0) = 0$ . Then prove that  $V$  and  $V - f$  are increasing functions on  $[a, b]$ .
20. Suppose  $\alpha$  increases monotonically on  $[a, b]$ ,  $g$  is continuous and  $g(x) = G'(x)$  for  $a \leq x \leq b$ . Prove that 
$$\int_a^b \alpha(x)g(x)dx = G(b)\alpha(b) - G(a)\alpha(a) - \int_a^b Gd\alpha.$$
21. Establish the existence of a real valued continuous function which is nowhere differentiable.
22. Prove that, for every interval  $[-a, a]$  there exists a sequence of real polynomials  $P_n$  such that  $P_n(0) = 0$  and such that  $\lim_{n \rightarrow \infty} P_n(x) = |x|$  uniformly on  $[-a, a]$ .  
(2×5=10 weightage)

