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Reg. No..............................

Name..................................

M.Sc. DEGREE (C.S.S.) EXAMINATION, MAY 2020

Fourth Semester

Faculty of Science

Branch I-(A)—Mathematics

MT04 E02—COMBINATORICS

(2012 Admission onwards)

Time : Three Hours | Maximum Weight : 30

Part A

Answer any **five** *questions.*
Each question has weight 1.

1. (a) Find the number of diagonals of a 80-sided polygon.

   (b) If C $(n, 8)$ = C $(n, 6)$ find C $(n, 2)$.

2. In how many ways can $n + 1$ different prizes to be awarded to n students in such a way that each student has at least one prize ?

3. Explain the Piegeon principle with an illustration.

4. Prove that R $(4, 4) \leq 18$ and R $(3, 4) \leq 9$.

5. Show that for $n \in$ N with $n \leq 3$, $\varphi(n)$ is always even.

6. State the principle of inclusion and exclusion.

7. Differentiate between ordinary generating function and exponential generating function.

8. Outline the method of solving a recurrence relation.

$(5 \times 1 = 5)$

Part B

Answer any **five** *questions.*
Each question has weight 2.

9. Give both algebraic and combinatorial proof for $\binom{n+1}{r} = \binom{n}{r-1} + \binom{n}{r}$.

Turn over

10. A committee of 8 is to be selected out of 6 males and 8 females. How many ways can be there so that males may not be outnumbered ?

11. For all integers $p, q \geq 2$ let $R(p-1, q)$ and $R(p, q-1)$ be even. Prove that

$$R(p, q) \leq R(p-1, q) + R(p, q-1) - 1.$$

12. (a) Explain generalised Piegeonhole principle with an example.

    (b) Write a note on the origin of Ramsey numbers.

13. Find the number of non-negative integer solutions of $x_1 + x_2 + x_3 = 12$ where $x_1 \geq 5, x_2 \geq 6, x_3 \geq 7$.

14. Define Euler function. Obtain a formula to compute it.

15. Solve $a_n - 5a_{n-1} + 6a_{n-2} = 2$ given $a_0 = 1$ and $a_1 = -1$.

16. In how many ways can four letters of the word EXAMINATION be arranged ?

(5 × 2 = 10)

Part C

Answer any **three** *questions.*
Each question has weight 5.

17. (a) How many numbers lying between 3000 and 4000 and divisible by 5 can be formed with the digits 3, 4, 5, 6, 7 and 8 repetition of the digits are not allowed ?

    (b) Find the number of r element multi-subsets of a set containing n elements.

18. (a) Let S be the set of natural numbers whose digits are chosen from {1, 3, 5, 7} such that no digits are repeated. Find $|S|$ and $\sum_{n \in s} n$.

    (b) Solve $a_n = \sqrt{3}\ a_{n-1} - a_{n-2}$ for $n \geq 2$ where $a_0 = 3$ and $a_1 = 2$.

19. State the problem of Tower of Hanoi and solve it using a recurrence relation.

20. Let ABC be an equilateral triangle and $\in$ the set of all points contained in the 3 line segments AB, BC, CA (including A, B, C). Show that for every partition $\in$ into 2 disjoint subsets, at least one of the 2 subsets contains the varities of a right angled triangle.

21. Prove $S(r, r-1) = \binom{r}{2}$ and $S(r, n) = S(r-1, n-1) + n\, S(r-1, n)$; $r, n \in N$ with $r \geq n$.

22. Explain the Sieve of Eratosathenes algorithm. Use it to find all prime numbers less than or equal to 50.

(3 × 5 = 15)