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B.C.A. DEGREE (C.B.C.S.S.) EXAMINATION, NOVEMBER 2017

First Semester

MATRICES, CALCULUS AND LAPLACE TRANSFORMS

(Complementary Mathematics for B.C.A.)

[2013–2016 Admissions]

Time : Three Hours   Maximum Marks : 80

Part A (Short Answer Questions)

Answer all questions.
Each question carries 1 mark.

1. Define a Hermitian matrix.

2. What do you mean by an inconsistent system of equations ?

3. Define a principal minor of a matrix.

4. Give the formal definition of limit of a function.

5. Find $\lim\limits_{x \to 2} \dfrac{(x-2)^2}{x^2-4}$.

6. Derive a partial differential equation by eliminating the constants from the equation :

   $a(x+y)+6(x-y)+c=z.$

7. Define complete and particular integrals of a partial differential equation.

8. Find the partial differential equation by eliminating the arbitrary function from $z = g\left(x+y^2\right)$.

9. Using the definition of Laplace transform find L $(\sin at)$.

10. Find the Laplace inverse of $\dfrac{1}{s\left(s^2+a^2\right)}$.

$(10 \times 1 = 10)$

Turn over

Part B (Brief Answer Questions)

Answer any eight questions.
Each question carries 2 marks.

11. Find the rank of $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 5 & 7 \end{bmatrix}$.

12. If $A = \begin{pmatrix} a & b \\ -b & a \end{pmatrix}$, $B = \begin{bmatrix} x & y \\ -y & x \end{bmatrix}$ if a, b, x, y are all different from zero find the inverse of A, B and verify that $(A \, B)^{-1} = B^{-1}A^{-1}$.

13. Find the eigen values and corresponding eigen vectors of the matrix $A = \begin{bmatrix} 1 & -2 \\ -2 & 4 \end{bmatrix}$.

14. Find the tangent line to the curve $f(x) = x + \dfrac{9}{x}$, at $x = -3$.

15. The curve $y = ax^2 + bx + c$ passes through the point $(1, 2)$ and is tangent to the line $y = x$ at the origin, find a, b and c.

16. Find the absolute maximum and minimum value of the function $f(x) = \sqrt[3]{x}$ where $-1 \le x \le 8$.

17. Form the partial differential equation by eliminating the functions from $z = f(x) + e^y g(x)$.

18. Solve $\dfrac{\partial^2 z}{\partial x \, \partial y} = \dfrac{x}{y} + a$.

19. Find the Laplace transform of $\cos 2t \cos 3t$.

20. Apply the convolution theorem to evaluate $L^{-1}\left(\dfrac{s}{s^2 + a^2}\right)$.

21. Find the inverse Laplace transform of $\dfrac{s+3}{s^2 - 4s + 13}$.

22. If $L\{F(t)\} = f(s)$ and $G(t) = \begin{cases} F(t-a) & t > a \\ 0 & t < a \end{cases}$ then prove that $L\{G(t)\} = e^{-as}f(s)$.

$(8 \times 2 = 16)$

Part C (Short Essay Questions)

Answer any six questions.
Each question carries 4 marks.

23. Obtain the row equivalent Canonical matrix C of the matrix A and hence find its rank, where $A = \begin{cases} 1 & 2 & -1 & 4 \\ 2 & 4 & 3 & 4 \\ -1 & -2 & 6 & -7 \end{cases}$.

24. Find the adjoint matrix of the matrix $D = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$.

25. State the mean value theorem and interpret it geometrically.

26. Let f be differentiable at every value of x and suppose that $f(1) = 1$, $f'(1) < 0$ on $(-\infty, 1)$, and $f' > 0$ on $(1, \infty)$. Show that $f(x) \ge 1$ for all x.

27. Solve $\dfrac{\partial^2 z}{\partial x^2} + z = 0$ given that when $x = 0$, $z = e^y$ and $\dfrac{\partial z}{\partial x} = 1$.

28. Solve $(x^2 - y^2 - z^2) p + 2xy \, q = 2xz$.

29. Find the differential equation of all planes which are at a constant distance 'a' from the origin.

30. Find the Laplace transform of :

$$\left(\sqrt{t} - \dfrac{1}{\sqrt{t}}\right)^3.$$

31. Apply convolution theorem to evaluate the inverse Laplace transform of $\dfrac{1}{\left(s^2 + a^2\right)^2}$.

$(6 \times 4 = 24)$

Part D (Long Essay Type Questions)

Answer any two questions.
Each question carries 15 marks.

32. Consider the system of equations :

$x + 2y + z = 2$, $3x + y - 2z = 1$

$4x - 3y - z = 3$, $2x + 4y + 2z = 4$.

(a) Check whether the given system is consistent or not.

(b) Solve the system of equations by using matrix method.

(c) Given $A = \begin{bmatrix} 2 & 3 & 6 \\ 3 & -6 & 2 \\ -6 & -2 & 3 \end{bmatrix}$. Find A^{-1} and A^TA, what type of a matrix is A ?

33. A dynamite blast blows a heavy rock straight up with a velocity of 160 ft/sec. It reaches a height of $s = 160\,t - 16\,t^2$ ft, after t sec, then :

    (a) How high does the rock go ?

    (b) What is the velocity and speed of the rock when it is 256 ft above the ground on the way up ? on the way down ?

    (c) What is the acceleration of the rock at any time 't' during its flight (after the blast) ? and when does the rock hit the ground again.

34. (a) Solve $\dfrac{\partial^2 z}{\partial x^2} = a^2 z$ given that when $x = 0$ $\dfrac{\partial z}{\partial x} = a\sin y$ and $\dfrac{\partial z}{\partial y} = 0$.

    (b) Solve $\left(x^2 - yz\right)p + \left(y^2 - zx\right)q = z^2 - xy$.

    (c) Find the differential equation of all spheres of fixed radius having their centres lie or the z-axis.

35. (a) If $L[f(t)] = \bar{f}(s)$ show that :

$L\left[(\sinh at)f(t)\right] = \tfrac{1}{2}\left[\bar{f}(s-a) - \bar{f}(s+a)\right]$ and $L\left[(\cosh at)f(t)\right] = \tfrac{1}{2}\left[\bar{f}(s-a) + \bar{f}(s+a)\right]$.

    (b) Find the inverse Laplace transform of $\dfrac{s}{s^4 + 4a^4}$.

    (c) Apply the convolution theorem to evaluate :

        (i) $L^{-1}\left(\dfrac{s}{\left(s^2 + a^2\right)^2}\right)$.

        (ii) $L^{-1}\left(\dfrac{s^2}{\left(s^2 + a^2\right)\left(s^2 + b^2\right)}\right)$.

$(2 \times 15 = 30)$